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Oblique Impact of a Jet on a Plane Surface

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IV. Oblique impact of a jet on a plane surface

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The dynamics of the region where a jet, striking a plane surface obliquely, is transformed into a thin sheet will be discussed. The maximum (stagnation) pressure is the same for all angles of incidence but the area over which the high pressure acts is much reduced as the angle of incidence, θ, becomes small. The main transformation from a jet to a sheet is due to a pressure of order $\sin^2\theta \times$ the stagnation pressure acting over an area of order $\operatorname{cosec}\theta \times$ the jet cross section. The pressure is due to the destruction of the component of velocity normal to the impact surface, but since pressure acts equally in all directions it imparts lateral velocity to streams which are not in the plane of symmetry. Each element of the stream can be regarded as passing through a small region where an impulsive body force changes its direction without changing its velocity, and some properties of this impulse will be described.

One case will be given where the transformations from a jet to a thin flat sheet can be described completely and the calculated distribution of pressure in the region where it occurs compared with experimental measurements.

Though a jet cannot produce a pressure greater than the stagnation pressure as a steady state, it seems theoretically possible to attain a much higher pressure for a short time when a very oblique jet is moved sideways.

When a jet strikes a flat plate it is transformed into a sheet which flows outwards radially over the plate from a region of impact whose dimensions are of the order of the cross section of the jet. A complete discussion of the mechanics of the situation would include friction effects at the solid surface, but in most cases this is relatively unimportant and it is sufficiently realistic to discuss the flow as though it were produced by the impact of two equal jets. In that case the plane of symmetry replaces the impact plate and there is no friction on that plane.

The physical event which occurs in the impact region is that a pressure is built up which serves to deflect streamlines from the direction of the jet to lines in the impact plane which spread out radially from the impact region. The stream velocity varies in passing through this region but, if the fluid is inviscid, regains its original value as it passes out of it. The overall effect on a stream tube of passing through the impact region is that which would be produced by a steady force applied in a direction that bisects the angle between its original and final directions.

Two dimensional jets

In general it has not been possible to calculate the distribution of pressure in the impact region but a complete solution was given by Michell (1890) for the impact of a flat jet in two dimensional flow. Without knowing the details of what happens in the impact region, one can show that a jet impinging obliquely on a plane at angle θ will divide itself into two, a fraction $\cos^2\frac{1}{2}\theta$ going forward and $\sin^2\frac{1}{2}\theta$ going backwards. This is a consequence of the conservation of momentum parallel to the plate. To find how the pressure is distributed over the plate it is necessary to use Michell's analysis. He did not give the necessary expressions in explicit form but I find that u_1 the nondimensional velocity on the plate, i.e. velocity/U (U being the velocity of the jet) is related to a nondimensional coordinate x_1 on the plate through an auxiliary variable q by the relations

$$u_1 = \frac{-1-q\cos\theta+\sqrt{(1-q^2)}\sin\theta}{q-\cos\theta}, \tag{1}$$

$$x_1 = \tfrac{1}{2}\{(1+\cos\theta)\ln(1+q)-(1-\cos\theta)\ln(1-q)\}+\sin\theta\sin^{-1}q+\text{const.}, \tag{2}$$

and the pressure p at x_1 is $$p = \tfrac{1}{2}\rho U^2(1-U_1^2). \tag{3}$$

Figures 1 and 2 show the distribution of $p/\frac{1}{2}\rho U^2$ as a function of $x_1 = \pi x/(\text{width of jet})$ for $\theta = 90°$ and $\theta = 30°$. The areas of the curves in figure 1 and the upper part of figure 2 represent the force that the jet exerts on the plate, which is $\rho U^2 \sin\theta$ (width of jet). Since $\sin 30° = \frac{1}{2}$, the area of the pressure curve in figure 1 is half that of figure 2.

In figure 1 and the upper part of figure 2 the width of the jet is marked and in the lower half of figure 2 the shape of the jet calculated from Michell's equations is also shown. It will be seen that when $\theta = 30°$ the jet divides into two portions so that only a fraction $\sin^2 15°$ or 6·7 % of it goes backwards. It will be noticed that though the downward force exerted on the plane decreases as θ decreases the maximum value of the pressure, namely $\frac{1}{2}\rho U^2$, remains unchanged but when θ becomes very small the area over which the pressure is near $\frac{1}{2}\rho U^2$ becomes very small.

Impact of a jet when flow is not confined to two dimensions

No complete description of the impact region analogous to Michell's has been given for any three dimensional case and it can be proved that considerations of conservation of momentum alone are incapable of giving the angular distribution of the thickness of the outflowing sheet formed by the impact. If we define the angular coordinate of a radius vector from the impact region on the impact plane as ϕ, and take $\phi = 0$ as the line of the projection of the inflowing jet on the plane, the proportion of the fluid which flows out between ϕ and $\phi+\mathrm{d}\phi$ is $F(\theta, \phi)$ and by definition

$$\int_0^{2\pi} F(\theta, \phi)\,\mathrm{d}\phi = 1. \tag{4}$$

$F(\theta, \phi)$ can be represented by a Fourier series

$$F(\theta, \phi) = \sum_0^\infty A_n \cos n\phi + \sum_1^\infty B_n \sin n\phi, \tag{5}$$

and since the jet is symmetrical in section, $B_n = 0$. The inward rate of flow of inertia parallel to any direction ϕ_0 is $\rho U^2 \cos\theta\cos\phi_0$ per unit area of cross section. The rate of outward flow in this direction is

$$\rho U^2 \int_0^{2\pi} F(\theta, \phi)\cos(\phi-\phi_0)\,\mathrm{d}\phi. \tag{6}$$

Since the pressure of the plate on the stream cannot contribute to this,

$$\cos\theta\cos\phi_0 = \int_0^{2\pi}\left(A_0+A_1\cos\phi+\sum_2^\infty A_n\cos n\phi\right)(\cos\phi\cos\phi_0+\sin\phi\sin\phi_0)\,\mathrm{d}\phi. \tag{7}$$

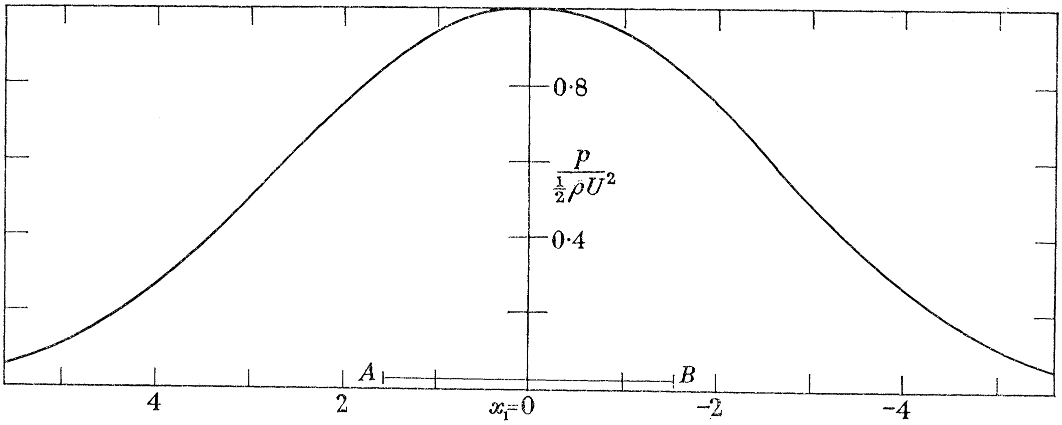

FIGURE 1. Pressure on plate, $\theta = 90°$. AB is jet width.

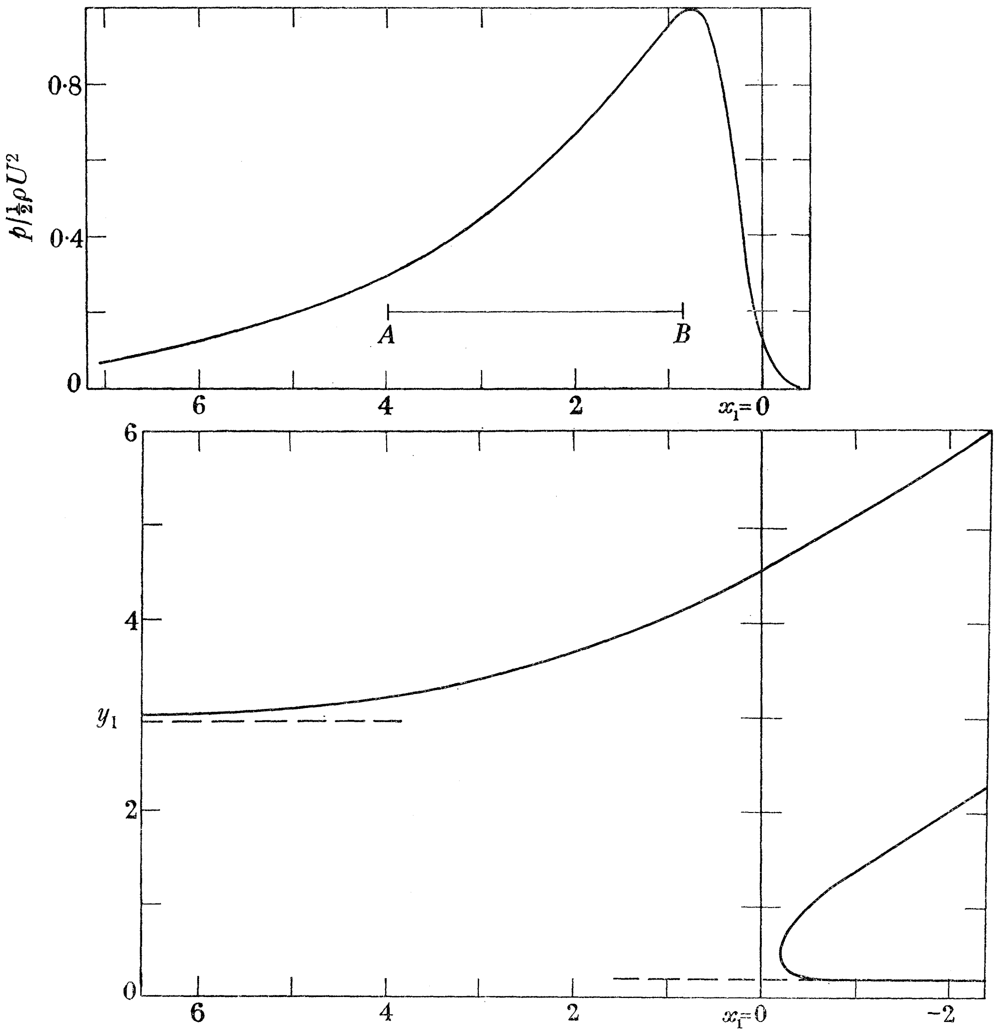

FIGURE 2. The upper half: pressure on plate $\theta = 30°$, AB is jet width.
The lower half: shape of jet when $\theta = 30°$.

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This equation is satisfied for all values of ϕ_0 provided $A_1 = (1/\pi)\cos\theta$. The only other condition is that to satisfy (6), $A_0 = 1/2\pi$. Thus for all values of $n \geqslant 2$ the coefficient A_n can have any values so far as the equations for inertia and continuity are concerned.

Though there seems to be no way in which $F(\theta, \phi)$ can be determined without making a complete analysis of the flow in the impact region it is worthwhile to look at the inverse problem and see what can be learned about the forces in the impact region if the value of $F(\theta, \phi)$, which can be measured, were known. Some years ago I measured $F(\theta, \phi)$ for the sheet produced by pairs of equal jets of circular cross section impinging at angles of $2\theta = 60$, 90 and 120°. In the paper (Taylor 1960) which described the measurements I gave the equations from which the component of force parallel to the impact plane which was needed to deflect each element of the jet into its observed angular position could be calculated. If $\rho U^2 I(\theta, \chi)\,\mathrm{d}\chi$ is the force required, I gave the relation between ϕ and χ, namely

$$\tan\chi = \frac{\sin\phi}{\cos\phi - \cos\theta}, \tag{8}$$

and also the formulae relating $F(\theta, \phi)$ to $I(\theta, \chi)$. The distribution of $I(\theta, \chi)$ round the plane must be such that all the forces are in equilibrium because every element in the set is balanced by the reaction of the rest. In other words, if a set of masses proportional to $I(\theta, \chi)$ were distributed on a circle their centre of gravity would be at the centre.

The measured values of $F(\theta, \phi)$ as a function of ϕ and the values of $I(\theta, \chi)$ calculated from them were given in figures 7 to 9 of Taylor (1960). Among the things I noticed but could not find any theoretical reason for was the fact that the $I(\theta, \chi)$ curve appeared to be very nearly symmetrical about $\chi = \frac{1}{2}\pi$. This was particularly remarkable because of the very great asymmetry of the $F(\theta, \phi)$ curves. Indeed this was so great that $I(\theta, 0)$ was almost exactly equal to $I(\theta, \pi)$, although $F(\theta, 0)$ for $\theta = 30°$ was 100 times as great as $F(\theta, \pi)$. This may be compared to the two dimensional case where for $\theta = 30°$ the thickness of the forward-going stream (analogous to $F(\theta, 0)$) was about 14 times as great as that of the backward-going stream analogous to $F(\theta, \pi)$. If the distribution of reaction force is symmetrical about $\chi = \frac{1}{2}\pi$ as well as $\chi = 0$ the force in every angular sector $\mathrm{d}\chi$ is exactly balanced by an equal reaction in the same interval $\mathrm{d}\chi$ but oppositely directed.

Another point I noticed was that the total reaction force between the two halves of the stream across the plane of symmetry (i.e. the force which gives rise to the lateral spread) was 0·64, 0·65 and 0·66 times the vertical force $\rho U^2 \sin\theta$ when θ was 30, 45 and 60°. The reaction across any axial plane when a round jet strikes a plane at right angles ($\theta = 90°$) can be shown theoretically to be $2/\pi = 0{\cdot}64$. The significance of this ratio (approximately $2/\pi$) probably lies in the fact that the jet was circular in section. It might be interesting to measure $F(\frac{1}{2}\pi, \phi)$, i.e. the angular distribution of thickness of the sheet produced by two jets of elliptic cross section aimed directly at one another.

Transformation of a converging into a diverging jet

The only case where I have been able to calculate the flow and distribution of pressure in a region where a diverging sheet is formed is that of a converging jet of elliptical cross section. The analysis for this case is given in Taylor (1960) and the experimental arrangements for producing the jet and for measuring the pressure in the region where the converging jet is transformed into a diverging jet are also described. The agreement between calculated and measured distribution of pressure along the axis is striking. Unfortunately there is a misprint in the description of the photograph of the jet (figure 4, Taylor 1960) where the scale length is given as 10 cm instead of 1·0 cm.

When this jet is converging the flow is somewhat analogous to two jets converging but the small high pressure region, which always occurs when the angle of convergence of two separate jets is small, does not occur in that case.

Theoretical possibility of the existence of a region where the maximum pressure can be much greater than $\frac{1}{2}\rho U^2$

Consider what happens when a jet of velocity U is directed downwards at a small angle θ on to a horizontal plane. Suppose that the horizontal plane now moves upwards with constant velocity V, the orifice remaining fixed in space. The impact plane can be brought to rest by moving the whole frame of reference downwards with velocity V. This of course makes the orifice also move downwards with velocity V. If now the whole frame of reference is given a velocity $V \cot \theta$ parallel to the plane, the flow is reduced to a steady state in which, relative to the new frame of reference, the orifice approaches the impact point with velocity $V \operatorname{cosec} \theta$. Thus in this frame of reference a steady stream approaches the impact point with velocity $U + V \operatorname{cosec} \theta$ and the pressure distribution is the same as it was before except that its magnitude is increased in the ratio $(1 + (V/U) \operatorname{cosec} \theta)^2$ and the maximum pressure is $\frac{1}{2}\rho U^2 (1 + (V/U) \operatorname{cosec} \theta)^2$. Suppose for instance that a jet at a head of 1 m of water is directed vertically downwards at a plate the maximum pressure is that of 100 cm of water and the jet velocity is 4·4 m/s. Suppose now that the plate is moved upwards with velocity 50 cm/s the maximum head of water is increased to $100(1 + 1/8{\cdot}8)^2 = 124$ cm of water. If now the jet is at 5° to the plane and it is moved upwards at 50 cm/s the maximum head will be $100(1 + \operatorname{cosec} \theta/8{\cdot}8)^2 = 730$ cm. This high pressure spot will of course move very rapidly across the impact plane.

References (Taylor)

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